Shadow Price & Sensitivity Analysis

Interpreting Solver outputs

Susana Barreiro

3 March 2021

Shadow prices

Max $Z = 3x_1 + 5x_2$

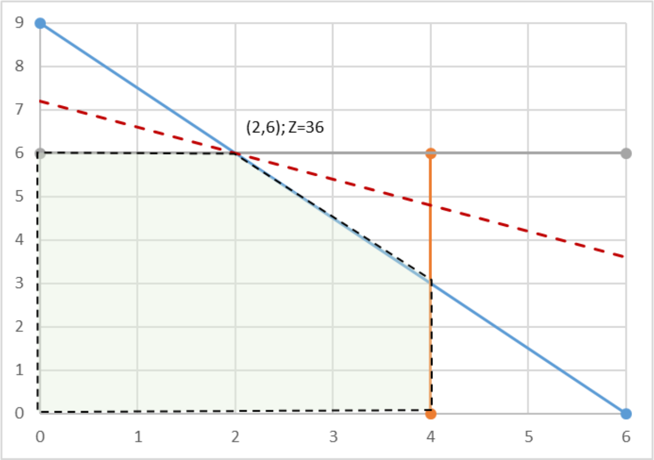
Subject to:

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

And $x_1 \geq 0; \quad x_2 \geq 0$



X1 number of window batch; X2 number of glass doors batch
Profit of windows batch = 3 profit of doors batch = 5 (K€)

Plant 1- produces the aluminum frames (prod. time available = 4 h/week)
Plant 2- produces the wood frames (prod. time available = 12)
Plant 3- produces the glass and assembles the product (prod. time available = 18)

Resources - the production capacity of each Plant made available (R1, R2, R3), where *bi (RHS)* represents the hours of production time per week

The Excel Formulation

			Max		
	Objective function:				
	x1	x2	Z		
	3	**5**	0		
	Constraint coeff.		Total		RHS
S1	**1**		0	<=	4
S2		**2**	0	<=	12
S3	**3**	**2**	0	<=	18
	x1	x2			
	0	0		>=	0

Shadow prices

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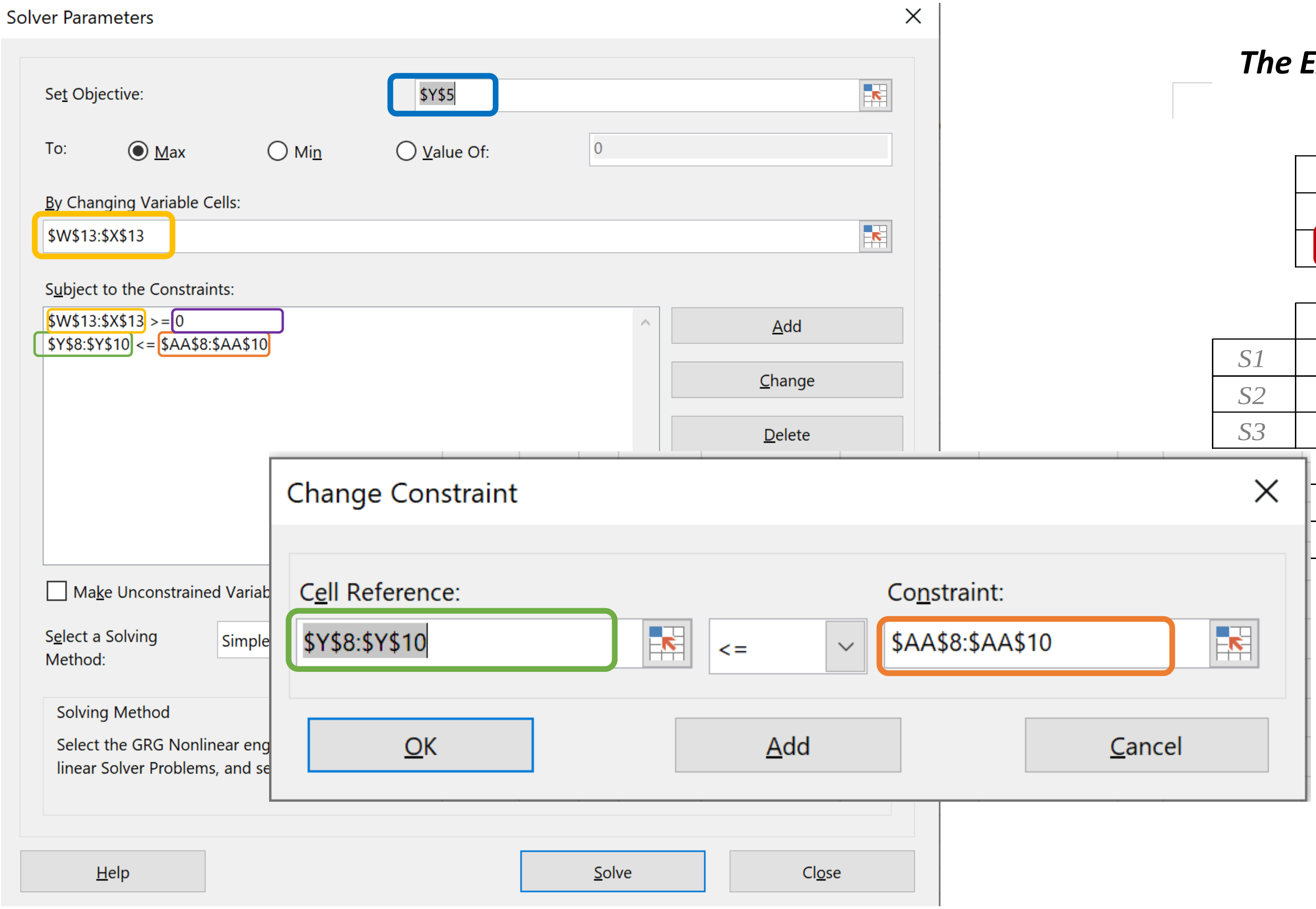
An example of how to solve this LP problem in Excel



The Excel Formulation

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S1	**1**		0	<=	4
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S3	**3**	**2**	0	<=	18

x1	x2		
0	0	>=	0

Because all the constraint signs are the same, constraint coeff. and their respective RHS can be selected in one step

When each of the constraints has different signs, these must be added one by one.

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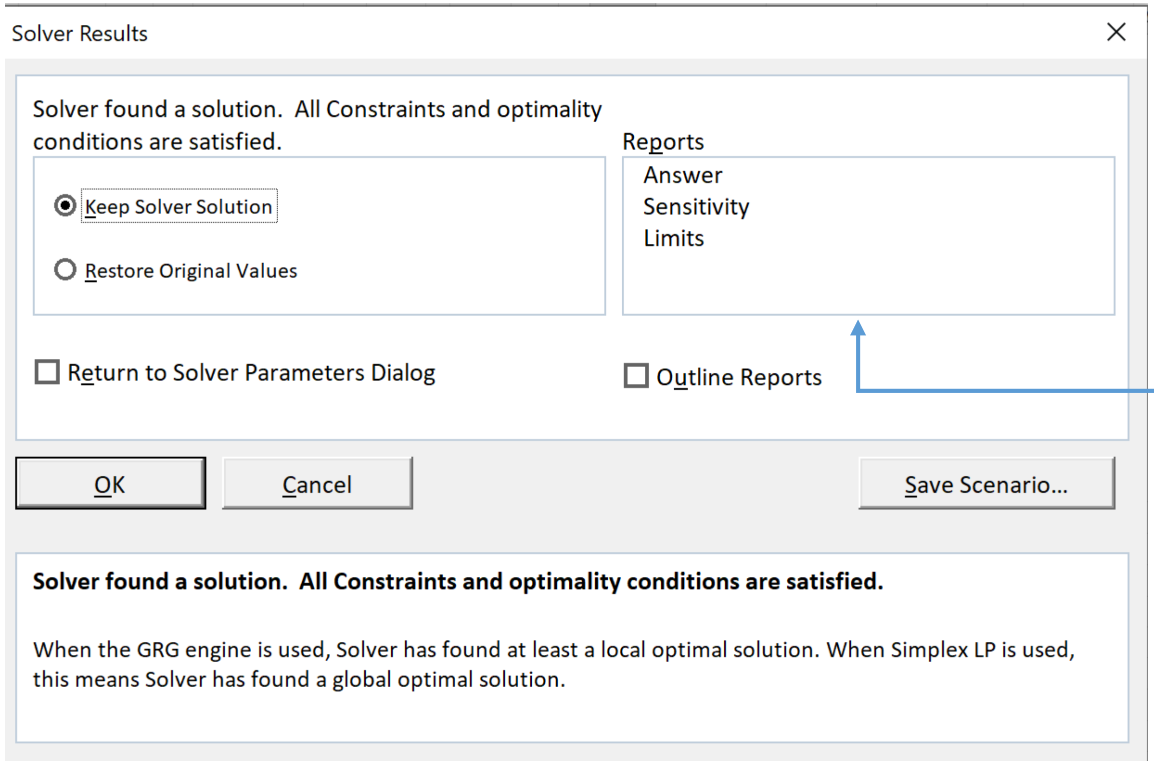

There is information we can obtain from the optimal tableau that we don't get directly in the Excel spreadsheet, but further details can be obtained by clicking on the option *Answer* under *Reports.*

Shadow prices

Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$Y$11	Total	0	36

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$W$4	S3 x1	0	2	Contin
$X$4	S3 x2	0	6	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Y$7	S1 Total	2	$Y$7<=$AA$7	Not Binding	2
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The Analysis Report indicates:

- The ***Objective Cell*** table tells us the starting value of the objective function (Z) when *Solver* was applied and the optimal value after *Solver*

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For more detailed information e.g (the shadow prices) a different option of the Reports should be selected: ***Sensitivity Analysis***

Sensitivity Analysis

The basic idea of **Sensitivity Analysis** is to be able to give answers to questions such as:

1. If the objective function changes, how does the solution change?
2. If resources available change, how does the solution change?
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(We will just focus on the first 2)

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Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $Z = \boxed{3x_1 + 5x_2}$

Subject to:

$$\begin{aligned} x_1 &\le 4 \\ 2x_2 &\le 12 \\ 3x_1 + 2x_2 &\le 18 \end{aligned}$$

$x_1, x_2 \ge 0$

Variable Cells

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First, let us analyze the ***Variable Cells*** part of the table:

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- The ***allowable increase and decrease*** show how much the ***coeff. of the objective function*** can change before the ***optimal solution*** has to be altered

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Since the ***Allowable Increase*** for X1 is 4.5 this means that if we increase the objective function coeff. for x1 up to an **Upper Limit** of **7.5** the optimal solution will not change (2, 6)

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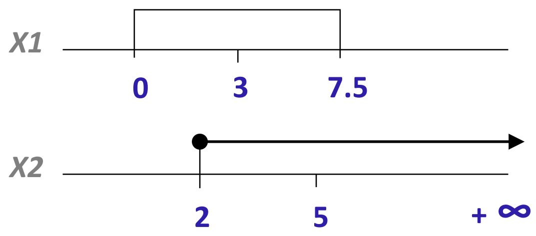
Excel usually represents very big numbers by **1E+30** which can be seen as **infinity**

Sensitivity Analysis



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So, what will happen if the **coeff. of X1 increases to 10** ?

- It will fall outside the allowable interval, thus the optimal solution will change (**Final Values)**

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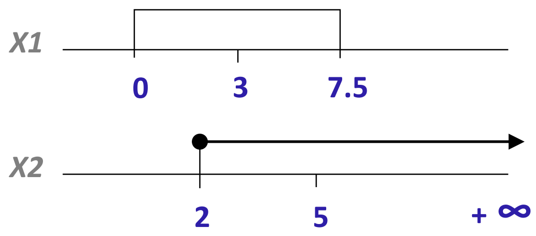

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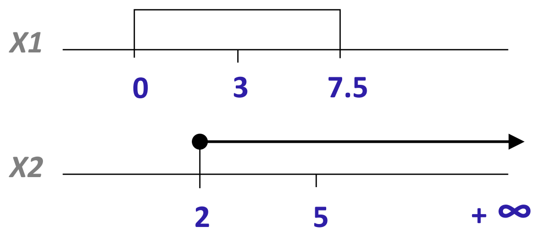
So, what will happen if the **coeff. of X1 increases to 10** ?

- It will fall outside the allowable interval, thus the optimal solution will change (**Final Values)**

And what will happen if the **coeff. of X1 increases to 6** ?

- The optimal solution will remain optimal but Z = **6***2 + **5***6 = 42

Sensitivity Analysis



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First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to $Z = \mathbf{3}*2 + \mathbf{5}*6 = 36$

And what will happen if **both coeff. X1 and X2 change to 4** (simultaneous changes)?

- This optimality report only applies to individual changes and to answer the question we will have to calculate **100% Rule**:

X1 increases in 1 unit, so: **1** / 4.5 (allowable increase) = 0.22

X2 decreases in 1 unit, so: 1 / 3 (allowable decrease) = 0.33

0.22 + 0.33 = 0.55 %
<100%
Solution remains optimal
$Z = \mathbf{4}*2 + \mathbf{4}*6 = 32$

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Reduced Cost column is set to zero for both variables because both products are being produced (2 units of X1 and 6 units of X2).

However, there might be situations for which not producing one of the products is more profitable (**Final Value** = 0). In such situations, the **Reduced Cost** = certain negative amount (for a maximization problem), which represent the reduction in profit that would be obtained if we insisted in producing one unit of that product

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Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

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Increase in Z resulting of an Unit increase in the RHS of a constraint

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Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

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$Y$8	S2 Total	12	1.5	12	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) without having to apply Simplex again as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$W\$4	S3 x1	2	0	3	4.5	3
\$X\$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
\$Y\$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
\$Y\$8	S2 Total	12	1.5	12 + 5 = 17	6	6	18	6
\$Y\$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we **increase** the **RHS** of constraint 2 by **5** (from 12 to 17):

5 * **1.5** = **7.5** , thus **Z = 36 + 7.5** = **43.5**

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12 - 5 = 7	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we **decrease** the **RHS** of constraint 2 by **5** (from 12 to 17):

- 5 * **1.5** = **7.5** , thus **Z = 36 - 7.5** = **28.5**

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12	6	6	18	6
$Y$9	S3 Total	18	$\boxed{1}$	18 + 7 = 11	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we decrease the **RHS** of constraint 3 by **7**, from 18 to **11**, please note that the Allowable Decrease is 6 making the **RHS** new value fall outside the **Lower Limit**. Therefore the **Shadow Price** is <u>no longer valid</u> and for that reason we <u>can not tell what would happen to profit.</u>