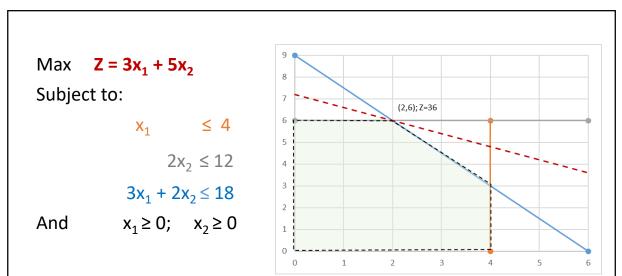
Shadow Price & Sensitivity Analysis

Interpreting Solver outputs

Susana Barreiro

3 March 2021

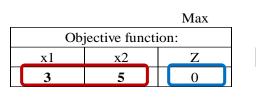


X1 number of window batch; X2 number of glass doors batch Profit of windows batch = 3 profit of doors batch = 5 ($K \in$)

Plant 1- produces the aluminum frames (prod. time available = 4 h/week) Plant 2- produces the wood frames (prod. time available = 12) Plant 3- produces the glass and assembles the product (prod. time available = 18)

Resources - the production capacity of each Plant made available (R1, R2, R3), where *bi* (*RHS*) represents the hours of production time per week

The Excel Formulation



	Constraint coeff.			Total	RHS				
<i>S1</i>		1			0	<=		4	
<i>S2</i>			2		0	<=		12	
<i>S3</i>		3	2		0	<=		18	

x1	x2
0	0

>=	0

An example of how to solve this LP problem in Excel

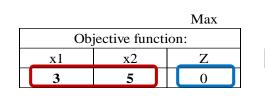
ver Parameters				×		
Se <u>t</u> Objective:	\$Y\$5					
To: <u>M</u> ax) Mi <u>n</u> (Value (Of:				
By Changing Variable Cells:						
\$W\$13:\$X\$13						
S <u>u</u> bject to the Constraints: \$W\$13:\$X\$13 >=0						
\$Y\$8:\$Y\$10 <= \$AA\$8:\$AA\$10			<u>A</u> dd			
			<u>C</u> hange			
			<u>D</u> elete			
	Change Constr	aint				
Ma <u>k</u> e Unconstrained Variab	C <u>e</u> ll Reference:			Co <u>n</u> st	raint:	
S <u>e</u> lect a Solving Simple Method:	\$Y\$8:\$Y\$10		<=	~ \$ AA\$	8:\$AA\$10	
Solving Method						
Select the GRG Nonlinear eng linear Solver Problems, and se	<u> </u>		<u>A</u> dd			<u>C</u> ancel

Max $Z = 3x_1 + 5x_2$ Subject to:

And

 $x_1 \le 4$ $2x_2 \le 12$ $3x_1 + 2x_2 \le 18$ $x_1 \ge 0; \quad x_2 \ge 0$

The Excel Formulation



[Constraint coeff.			Total]	RHS		
<i>S1</i>	1			0	<=	4		
<i>S2</i>		2		0	<=	12		
<i>S3</i>	3	2		0	<=	18		
~]		_					
Х	x1	x2						
	0	0			>=	0	J	

Because all the constraint signs are the same, constraint coeff. and their respective RHS can be selected in one step

When each of the constraints has different signs, these must be added one by one.

Max **Z = 3x**₁ + 5x₂

Subject to:

And

RHS 4

12

18

0

<=

<=

<=

 $\geq =$

≤ 4 X_1 $2x_2 \le 12$ $3x_1 + 2x_2 \le 18$ $x_1 \ge 0; \quad x_2 \ge 0$

							Ma
					Ob	jective funct	tion:
					x1	x2	Z
					3	5	0
olver Results		×					
			_		Constrai	int coeff.	Tot
Solver found a solution. All Constraints and optir conditions are satisfied.				<i>S1</i>	1		0
conditions are satisfied.	Re <u>p</u> orts Answer			<i>S2</i>		2	0
• Keep Solver Solution	Sensitivity			<i>S3</i>	3	2	0
	Limits			_			_
O <u>R</u> estore Original Values					x1	x2	
					0	0	
Return to Solver Parameters Dialog		T II					_
	O <u>u</u> tline Reports						
<u>O</u> K <u>C</u> ancel		Save Scenario	 .				
			There is	Into	rmation	we can	obtair
			tableau	that	we don'	't get dir	ectly
Solver found a solution. All Constraints and opt	imality conditions are satis	fied.	spreads			0	•
			spiedusi	ieet,	, but iui		

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

n from the optimal in the Excel an be obtained by clicking on the option *Answer* under *Reports*.

Max Time Unlimited, Iterations Unlimited, Precision 0.000001 Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Cell Name Original Value Final Value
\$Y\$11 Total 0 36

Variable Cells

Cell I	Name	Original Value	Final Value	Integer
\$W\$4 S3	x1	0		2 Contin
\$X\$4 S3	x2	0		6 Contin

The Analysis Report indicates:

- The *Objective Cell* table tells us the starting value of the objective function (Z) when *Solver* was applied and the optimal value after *Solver*

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$Y\$7	S1 Total		2 \$Y\$7<=\$AA\$7	Not Binding	2
\$Y\$8	S2 Total	1	2 \$Y\$8<=\$AA\$8	Binding	0
\$Y\$9	S3 Total	1	.8 \$Y\$9<=\$AA\$9	Binding	0
\$W\$4	S3 x1		2 \$W\$4>=0	Not Binding	2
\$X\$4	S3 x2		6 \$X\$4>=0	Not Binding	6

Max Time Unlimited, Iterations Unlimited, Precision 0.000001 Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Max)							
Cell	Name	Original Value	Final Value				
\$Y\$11	Total	0	36				

Va	riable (Cells				
	Cell	Name	Original Value	Final Value	Integer	
	\$W\$4	S3 x1	0	-	2 Contin	
	\$X\$4	S3 x2	0	6 Contin		

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\$Y\$7	S1 Total		2 \$Y\$7<=\$AA\$7	Not Binding	2
\$Y\$8	S2 Total	1	2 \$Y\$8<=\$AA\$8	Binding	0
\$Y\$9	S3 Total	1	8 \$Y\$9<=\$AA\$9	Binding	0
\$W\$4	S3 x1		2 \$W\$4>=0	Not Binding	2
\$X\$4	S3 x2		6 \$X\$4>=0	Not Binding	6

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Objectiv	ve Cell (Max	()	
Cel	l Name	Original Value	Final Value
\$Y\$1	1 Total	0	36

/ariable (Cells			
Cell	Name	Original Value	Final Value	Integer
\$W\$4	S3 x1	0	2	Contin
\$X\$4	S3 x2	0	6	Contin

Cell	Name	Cell Value	Formula	Status	Slack
\$Y\$7	S1 Total		2 \$Y\$7<=\$AA\$7	Not Binding	2
\$Y\$8	S2 Total		12 \$Y\$8<=\$AA\$8	Binding	(
\$Y\$9	S3 Total		18 \$Y\$9<=\$AA\$9	Binding	(
\$W\$4	S3 x1		2 \$W\$4>=0	Not Binding	2
\$X\$4	S3 x2		6 \$X\$4>=0	Not Binding	E

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- The initial and optimal solutions (x1, x2, S1, S2) can be read across tables

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Objective	Cell (Max)	
Cell	Name	Original Value	Final Value
\$Y\$11	Total	0	36

Cell	Name	Original Val	ue	Final Value	Integer	
\$W\$4	S3 x1		0	2	2 Contin	
\$X\$4	S3 x2		0	6	5 Contin	
onstrain Cell	ts Name	Cell Value		Formula	Status	Sl <mark>ack</mark>
		Cell Value		Formula \$Y\$7<=\$AA\$7	Status Not Binding	Sla ck 2
Cell	Name	Cell Value	2			
Cell \$Y\$7	Name S1 Total	Cell Value	2 12	\$Y\$7<=\$AA\$7	Not Binding	2
Cell \$Y\$7 \$Y\$8	Name S1 Total S2 Total S3 Total	Cell Value	2 12 18	\$Y\$7<=\$AA\$7 \$Y\$8<=\$AA\$8	Not Binding Binding	2 0

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Oł	ojective	Cell (Max)		
	Cell	Name	Original Value	Final Value
	\$Y\$11	Total	0	36

Cell	Name	Original Val	ue	Final Value	Integer		
\$W\$4	S3 x1		0	2	2 Contin		
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onstrain Cell	ts Name	Cell Value		Formula	Status	Sla	ick
		Cell Value		Formula \$Y\$7<=\$AA\$7	Status Not Binding	Sla	<mark>rek</mark> 2
Cell	Name	Cell Value	2			Sla	
Cell \$Y\$7	Name S1 Total	Cell Value	2 12	\$Y\$7<=\$AA\$7	Not Binding	Sla	2
Cell \$Y\$7 \$Y\$8	Name S1 Total S2 Total S3 Total	Cell Value	2 12 18	\$Y\$7<=\$AA\$7 \$Y\$8<=\$AA\$8	Not Binding Binding	Sla	2 0

For more detailed information e.g (the shadow prices) a different option of the Reports should be selected: **Sensitivity Analysis**

The basic idea of **Sensitivity Analysis** is to be able to give answers to questions such as:

1. If the objective function changes, how does the solution change?

- 2. If resources available change, how does the solution change?
- 3. If a constraint is added to the problem, how does the solution change?

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(We will just focus on the first 2)

The basic idea of **Sensitivity Analysis** is to be able to give answers to questions such as:

If the objective function changes, how does the solution change?
If resources available change, how does the solution change?
If a constraint is added to the problem, how does the solution change?

(We will just focus on the first 2)

Interpreting the *Solver Sensitivity Report*:

Max $Z = 3x_1 + 5x_2$ Subject to: $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_{1,}x_2 \geq 0$

		Final	Reduced	Objective	Allowable	Allowable
Cell	Name	Value	Cost	Coefficient	Increase	Decrease
\$W\$4	S3 x1	2	0	3	4.5	3
\$X\$4	S3 x2	6	0	5	1E+30	3
Constrair	nts					
Constrair		Final	Shadow	Constraint	Allowable	Allowable
<u> </u>	nts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Constrair						
Constrain Cell	Name	Value	Price	R.H. Side	Increase	Decrease

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_{1,}x_2 \geq 0$

First, let us analyze the *Variable Cells* part of the table:

- The Final Value = **Optimal Solution**, thus replacing the optimal (x1, X2) in the objective function leads to Z = 3*2 + 5*6 = 36

Va	riable (Cells					
			Final	Reduced	Objective	Allowable	Allowable
	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
	\$W\$4	S3 x1	2	0	3	4.5	3
	\$X\$4	S3 x2	6	0	5	1E+30	3
Со	nstrain	ts	Final	Shadow	Constraint	Allowable	Allowable
Со	nstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Со							
Со	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
Со	Cell \$Y\$7	Name S1 Total	Value 2	Price 0	R.H. Side 4	Increase 1E+30	Decrease 2

Interpreting the Solver Sensitivity Report:

Max **Z = 3x₁ + 5x₂**

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 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_1, x_2 \geq 0$

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- The *allowable increase and decrease* show how much the *coeff. of the objective function* can change before the *optimal solution* has to be altered

		Final	Reduced	Objective	Allowable	Allowable
Cel	Name	Value	Cost	Coefficient	Increase	Decrease
\$W\$	4 S3 x1	2	0	3	4.5	3
\$X\$4	S3 x2	6	0	5	1E+30	3
Constra	nts					
Constrai		Final	Shadow	Constraint	Allowable	Allowable
Constrai	nts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	
						Allowable
Cel	Name	Value	Price	R.H. Side	Increase	Allowable Decrease

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

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		Final	Reduced	Objective	Allowable	Allowable	Upper	Lower
Cell	Name	Value	Cost	Coefficient	Increase	Decrease	Limit	Limit
\$W\$4	S3 x1	2	0	3	+ 4.5	3	7.5	0
6V64	c_{2}	C	0	5	1E+30	3		
\$X\$4 Constrain	S3 x2	6	0	5	11+30	3	-	
Constrain	its	Final	Shadow	Constraint	Allowable	Allowable	-	
							-	
Constrain	its	Final	Shadow	Constraint	Allowable	Allowable	-	
Constrain Cell	its Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable	-	

Since the **Allowable Increase** for X1 is 4.5 this means that if we increase the objective function coeff. for x1 up to an **Upper Limit** of **7.5** the optimal solution will not change (2, 6)

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

First, let us analyze the *Variable Cells* part of the table:

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		Final	Reduced	Objective	Allowable	Allowable	Upper	Lowe
Cell	Name	Value	Cost	Coefficient	Increase	Decrease	Limit	Limit
\$W\$4	S3 x1	2	0	3	4.5	3	7.5	0
\$X\$4	S3 x2	6	0	5	1E+30	3	+ ∞	2
onstrain							-	L
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	-	L
<u> </u>								L
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	-	L
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	-	L

Since the **Allowable Increase** for X1 is 4.5 this means that if we increase the objective function coeff. for x1 up to an **Upper Limit** of **7.5** the optimal solution will not change (2, 6)

Excel usually represents very big numbers by **1E+30** which can be seen as **infinity**

Interpreting the Solver Sensitivity Report:

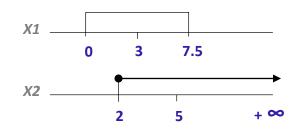
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/a <mark>riable (</mark>	Cells							
		Final	Reduced	Objective	Allowable	Allowable	Upper	Lower
Cell	Name	Value	Cost	Coefficient	Increase	Decrease	Limit	Limit
\$W\$4	S3 x1	2	0	3	4.5	3	7.5	0
\$X\$4	S3 x2	6	0	5	1E+30	3	+∞	2
Constrain	its							
Constrain	its	Final	Shadow	Constraint	Allowable	Allowable	-	
Co <u>nstrain</u> Cell	its Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	-	
							-	
Cell	Name	Value	Price	R.H. Side	Increase	Decrease	-	
Cell \$Y\$7	Name S1 Total	Value 2	Price 0	R.H. Side 4	Increase 1E+30	Decrease 2	-	

So, what will happen if the coeff. of X1 increases to 10?

- It will fall outside the allowable interval, thus the <u>optimal solution will</u> <u>change</u> (Final Values)

Interpreting the Solver Sensitivity Report:

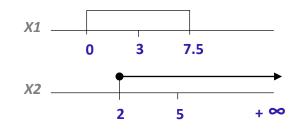
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		Final	Reduced	Objective	Allowable	Allowable	Upper	Lowe
Cell	Name	Value	Cost	Coefficient	Increase	Decrease	Limit	Limit
\$W\$4	S3 x1	2	0	3	4.5	3	7.5	0
\$X\$4	S3 x2	6	0	5	1E+30	3	+ ∞	2
<u> </u>			Shadow	Constraint	Allowable	Allowable	-	_
onstrain		Final	Shadow	Constraint	Allowable	Allowable	-	
<u> </u>			Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease		
onstrain	ts	Final					-	
onstrain Cell	ts Name	Final Value	Price	R.H. Side	Increase	Decrease	-	

So, what will happen if the coeff. of X1 increases to 10?

- It will fall outside the allowable interval, thus the <u>optimal solution will</u> <u>change</u> (Final Values)

And what will happen if the coeff. of X1 increases to 6?

- The optimal solution will remain optimal but Z = 6*2 + 5*6 = 42

Interpreting the Solver Sensitivity Report:

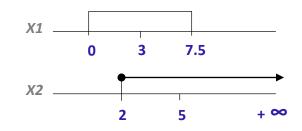
Max **Z = 3x**₁ + 5x₂

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		Final	Reduced	Objective	Allowable	Allowable	Upper	Lowei
Cell	Name	Value	Cost	Coefficient	Increase	Decrease	Limit	Limit
\$W\$4	S3 x1	2	0	4 - 3	= 1 4.5	3	7.5	0
\$X\$4	S3 x2	6	0	5	- 4 =1 1E+30	3	+ ∞	2
<u> </u>							-	2
<u> </u>		Final	Shadow	Constraint	Allowable	Allowable	-	L
<u> </u>							-	L
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	-	L
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable	-	L

And what will happen if **both coeff. X1 and X2 change to 4** (simultaneous changes)?

- This optimality report only applies to individual changes and to answer the question we will have to calculate **100% Rule**:

X1 increases in 1 unit, so: 1 / 4.5 (allowable increase) = 0.22 X2 decreases in 1 unit, so: 1 / 3 (allowable decrease) = 0.33 0.22 + 0.33 = 0.55 % <100% Solution remains optimal Z = 4*2 + 4*6 = 32

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

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\$W\$4	S3 x1	2	0	4 - 3	= 1 4.5	3
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Constrair			Shadow	Constraint	Allowable	
		Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Constrair	its	Final				Allowable
Constrair Cell	its Name	Final Value	Price	R.H. Side	Increase	Allowable Decrease

Reduced Cost column is set to zero for both variables because both products are being produced (2 units of X1 and 6 units of X2).

However, there might be situations for which not producing one of the products is more profitable (Final Value = 0). In such situations, the Reduced Cost = certain negative amount (for a maximization problem), which represent the reduction in profit that would be obtained if we insisted in producing one unit of that product

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_{1,}x_2 \geq 0$

Now, let us analyze the *Constraints* part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

		Final	Reduced	Objective	Allowable	Allowable		
Cell	Name	Value	Cost	Coefficient	Increase	Decrease		
\$W\$4	S3 x1	2	0	3	4.5	3		
\$X\$4	S3 x2	6	0	5	1E+30	3]	
onstrain							-	
<u> </u>		Final	Shadow	Constraint	Allowable	Allowable	Upper	Lowe
<u> </u>		Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	-	Lowe Limit
onstrain	ts						Upper Limit	
onstrain Cell	ts Name	Value	Price	R.H. Side	Increase	Decrease	Upper Limit 7.5	

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Subject to:

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onstrain							-	
<u> </u>		Final	Shadow	Constraint	Allowable	Allowable	Upper	Lowe
<u> </u>							Upper Limit	Lowe Limit
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable		
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Limit +∞	Limit

Increase in Z resulting of an Unit increase in the RHS of a constraint

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_1, x_2 \geq 0$

Now, let us analyze the *Constraints* part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged Optimal sol.: (x1, x2) =(0, 6) Z = 36

		Final	Reduced	Ob	ojective	Allowable	Allowable		
Cell	Name	Value	Cost	Coe	efficient	Increase	Decrease		
\$W\$4	S3 x1	2	0		3	4.5	3	3	
\$X\$4	S3 x2	6	0		5	1E+30	3	3	
onstrain	ts								
onstrain	ts	Final	Shadow	Со	nstraint	Allowable	Allowable	Upper	Lowe
onstrain Cell	ts Name	Final Value	Shadow Price		nstraint H. Side	Allowable Increase	Allowable Decrease	Upper Limit	
		-						Limit	
Cell	Name	Value	Price		H. Side	Increase	Decrease	$\frac{\text{Limit}}{+\infty}$	Lowe Limit 2 6

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Increase in Z resulting of an Unit increase in the RHS of a constraint

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Increase in Z resulting of an Unit increase in

the RHS of a constraint

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_1, x_2 \geq 0$

Now, let us analyze the *Constraints* part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged Optimal sol.: (x1, x2) =(0, 6) Z = 36

		Final	Reduced	Objective	Allowable	Allowable		
Cell	Name	Value	Cost	Coefficient	Increase	Decrease		
\$W\$4	S3 x1	2	0	3	4.5	3]	
\$X\$4	S3 x2	6	0	5	1E+30	3		
<u> </u>								
onstrain		Final	Shadow	Constraint	Allowable	Allowable	Upper	Lowe
<u> </u>								Lowe Limit
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	Upper Limit	
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side 4	Allowable Increase	Allowable Decrease	Upper Limit +∞	Limit

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the RHS of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we increase the RHS of constraint 2 by 5 (from 12 to 17):

5 * **1.5** = **7.5**, thus **Z** = **36** + **7.5** = **43.5**

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Increase in Z resulting of an Unit increase in

the RHS of a constraint

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_1, x_2 \geq 0$

Now, let us analyze the *Constraints* part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged Optimal sol.: (x1, x2) =(0, 6) Z = 36

		Final	Reduced	Objective	Allowable	Allowable		
Cell	Name	Value	Cost	Coefficient	Increase	Decrease		
\$W\$4	S3 x1	2	0	3	4.5	3		
\$X\$4	S3 x2	6	0	5	1E+30	3		
onstrain							_	
<u> </u>		Final	Shadow	Constraint	Allowable	Allowable	Upper	Lowe
<u> </u>							_	Lowe Limit
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	Upper Limit	
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side 4	Allowable Increase	Allowable Decrease	Upper Limit +∞	Limit

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we **decrease** the **RHS** of constraint 2 by **5** (from 12 to 17):

Interpreting the Solver Sensitivity Report:

Max **Z = 3x**₁ + 5x₂

Subject to:

 $x_1 \leq 4$ $2x_2 \leq 12$ $3x_1 + 2x_2 \leq 18$ $x_1, x_2 \geq 0$

Now, let us analyze the *Constraints* part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged Optimal sol.: (x1, x2) =(0, 6) Z = 36

		Final	Reduced	Objective	Allowable	Allowable		
Cell	Name	Value	Cost	Coefficient	Increase	Decrease		
\$W\$4	S3 x1	2	0	3	4.5	3		
\$X\$4	S3 x2	6	0	5	1E+30	3		
onstrain	ts						-	
onstrain	ts	Final	Shadow	Constraint	Allowable	Allowable	Upper	Lowe
onstrain Cell	ts Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lowe Limit
		-					Limit	
Cell	Name	Value	Price	R.H. Side	Increase	Decrease	Limit +∞	Limit

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we decrease the RHS of constraint 3 by 7, from 18 to 11, please note that the Allowable Decrease is 6 making the RHS new value fall outside the Lower Limit. Therefore the Shadow Price is <u>no longer valid</u> and for that reason we <u>can not tell what would happen to profit</u>.

Increase in Z resulting of an Unit increase in the RHS of a constraint